Modeling and Dynamic Simulation of Induction Machine under Mixed Eccentricity Conditions using Winding Function

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ABSTRACT
The winding function theory is normally used for analysis of eccentricity of induction machines. However, it seems that the derived equations for such analysis in the available literature are not quite correct. This paper reviews the main stages of the winding function derivation and this function is then extended to a non-uniform airgap case. Permeance of the non-uniform airgap for a mixed eccentricity condition is evaluated where a more accurate approximation compared to the available analysis is used. The linear rise of mmf in the airgap above the stator and rotor slots is taken into account in this calculation. It means that all spatial harmonics of the airgap field distribution are accounted for also, finite element computations results are compared to inductances, which obtained using the analytical results, and good agreements are achieved. Finally, using calculated inductances, performance of a typical induction machine and its phase frequency spectrum under mixed eccentricity condition are compared with that of the healthy motor.

1. Introduction
Winding function theory introduced for magnetic inductance evaluation of induction motors in 1965. The basic capability of this theory in evaluation of motor inductances with any stator and rotor winding distribution leads to many analysis of electrical machines such as single-phase [1], three-phase [2] and linear [3] induction machines and synchronous reluctance machine under different performance conditions from the normal to saturation [4] and internal faults [5] conditions. Since this theory calculates inductances of motor based on the distribution of stator and rotor winding, it is capable well to model different faults of the induction motor such as eccentric rotor, stator turn-to-turn fault, stator coil-to-coil fault, broken bar of rotor, broken yoke of rotor and even cracking of rotor bars. Thus, using this theory for analysis of motor performance under such conditions is easily possible and it is the main background of several studies in a faulty induction motors diagnosis [6, 7]. However, when the fault leads to a non-uniform airgap, application of this theory is more difficult; while this theory has been so far used for analysis of induction motor under stator winding and rotor cage faults, it has been also employed under stator and rotor eccentricity fault where a non-uniform airgap exists

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This paper reviews the winding function theory and extends it to a non-uniform airgap. This leads to a more comprehensive definition of the winding function. Then, accurate permeance of the airgap under mixed eccentricity is calculated and a more accurate approximation, compared to the available techniques, is used. Different inductances of motor with this modified winding function and approximation of the airgap permeance distribution are evaluated. Finally, the calculated inductances are used to analyze induction motor under a mixed eccentricity.

2. Induction Motor Modeling

The governing equations of three-phase squirrel-cage rotor induction motor are as follows:

$$[\psi_s] = [L_s][i_s] + [L_{w}] [i_r]$$

(1) $$[\psi_r] = [L_{r0}][i_s] + [L_{r}] [i_r]$$

(2) $$[V_s] = [R_s][i_s] + \frac{d}{dt}[\psi_s]$$

(3) $$[V_r] = [R_r][i_r] + \frac{d}{dt}[\psi_r]$$

(4) $$T_m = [i_s] \left( \frac{d}{dt} [L_{sr}] \right) [i_r]$$

(5) $$\frac{d}{dt} \omega_r = T_m - T_i$$

(6)

In these equations:

$$[F_s] = \left[ f_1 \ f_2 \ ... \ f_n \right]^T$$

(8) $$[F_r] = \left[ f'_1 \ f'_2 \ ... \ f'_n \right]^T$$

(9)

where F can be I, V or ψ. Considering the stator phases and rotor rings, matrices $[R_s]$ and $[R_r]$ are calculated as follows:

$$[R_s] = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix}$$

(10) $$[R_r] = [\theta]_{mn} : \begin{cases} r_y = 2(r_h + r_e) & i = j \\ r_y = -r_e & (i = j \pm 1) \ or \ (i = I_e, j = n) \ or \ (i = n, j = I) \\ r_y = 0 & \text{else} \end{cases}$$

(11)

3. Evaluation of Airgap Permeance under Eccentricity Conditions

Permeance of airgap is defined as follows:

$$P(\varphi) = \mu_0 \frac{r_{os}(\varphi)}{g(\varphi)}$$

(12)

Thus, for calculation of this quantity, the airgap distribution about rotor and its variation versus rotor angular position must be estimated. For a mixed eccentricity between rotor and stator, three symmetrical axes of stator, rotor and rotor rotation are displaced in parallel. Ratio of displacement of rotor rotation axes from the stator symmetry axis, uniform airgap length is called static eccentricity ($\delta_s$), ratio of displacement of rotor symmetry axis from its rotation axes, and uniform airgap length is called dynamic eccentricity ($\delta_d$). Angles of these two displacements in the stator reference frame are called the static ($\alpha_s$) and dynamic ($\alpha_d$) eccentricity angles. It can be shown that the static eccentricity angle is constant and equal to its initial value, but the dynamic eccentricity angle varies with changing the rotor angular position.

Fig. 1 shows the position of symmetry and rotation axes of the rotor in the stator reference frame when there are both static and dynamic eccentricity between rotor and stator. In this figure, the static eccentricity vector $O_sO_\omega$ is equal to $\delta_s$ with angle $\alpha_s$, and the dynamic eccentricity vector $O_rO_\omega$ is equal to $\delta_d$ with angle $\alpha_d$. Therefore, the mixed vector is defined as follows:

$$O_sO_\omega = O_rO_\omega + O_{sr}O_r$$

(13)

When the static eccentricity angle and initial dynamic eccentricity angle are equal to zero, the vector can be calculated as follows:

$$\delta = |O_sO_\omega| = \sqrt{\delta_s^2 + \delta_d^2 + 2\delta_s\delta_d \cos\theta}$$

(14) $$\alpha = \arctan \frac{\delta_d \sin \theta}{\delta_s + \delta_d \cos \theta}$$

(15)

Then the coordinates of rotor and stator symmetry axis may be obtained at any time. The precise length of
the airgap is:

\[ g(\varphi) = R_s - \delta g_0 \cos(\varphi - \alpha) - \sqrt{R_s^2 - \delta^2 - \delta^2 g_0^2 \sin(\varphi - \alpha)} \] (16)

and the radius of the airgap is:

\[ r_{av}(\varphi) = R_s - \frac{1}{2} g(\varphi) \] (17)

Permeance of the airgap can be evaluated based on Eqns. 12, 16, 17, as follows:

\[ F(\varphi) = \mu_0 \frac{g_0}{g_0} (A + B \cos(\varphi - \alpha) + C \cos(2\varphi - 2\alpha)) \] (18)

It is noted that so far two first terms of Eqn. 18 are used. Although it is a good approximation in the static and dynamic eccentricities conditions [8, 9], it is not so in the case of mixed eccentricity condition [11].

4. Winding Function Theory under Eccentricity

A general view on the winding function theory is a definition of a function for winding x such that the resultant distribution of magnetic field intensity of the winding is expressed in the product of this winding, winding current and inverse of the airgap distribution. Using the path shown in Fig. 2 and the Ampere's law for winding x, the following can be written:

\[ \oint \dot{H}_x (r, \varphi) d\ell = \oint S_x (r, \varphi) ds \] (20)

\[ \oint S_x (r, \varphi) ds = n_x (\varphi) i_x \] (21)

Since the relative permeability of iron is much larger than that of the airgap, mmf drop within the iron can be ignored in comparison with the airgap. Because of the small airgap length, it is also assumed that the magnetic intensity in an arbitrary angle \( \varphi \) is independent of the radius and equal to its value in the middle of the airgap, therefore:

\[ \oint \dot{H}_x (R_s) d\ell = \oint \dot{H}_{\text{airgap}} (R_s) d\ell = \dot{H}_{\text{airgap}} (\varphi) d\ell = \dot{H}_x (\varphi) g(\varphi) - \dot{H}_x (0) g(0) \] (22)

Combination of the above equations leads to:

\[ \oint \dot{H}_{\text{airgap}} (r, \varphi) d\ell = n_x (\varphi) i_x \] (23)

Calculation of the above integral yields:

\[ H_x (\varphi) = \frac{n_x (\varphi) i_x + H_x (0) g(0)}{g(\varphi)} \] (24)

In order to evaluate \( H_x(\varphi) \), the cylindrical Gaussian surface in the depth of the airgap is considered as follows:

\[ \oint_S \mu_0 \dot{H}_x (\varphi) ds = 0 \] (25)

Two latter equations yield:

\[ \oint_S n_x (\varphi) g(\varphi) i_x d\varphi + \oint_S g(0) \dot{H}_x (0) \frac{2\pi}{g(\varphi)} d\varphi = 0 \] (26)
Combination of Eqns. 24 and 26 gives:

\[
H_x(\varphi) = \left( n_x(\varphi) - \frac{\int_0^{2\pi} \frac{n_x(\varphi)}{g(\varphi)} d\varphi}{\frac{1}{g(\varphi)}} \right) \frac{I_x}{2\pi} \left( \int_0^{2\pi} \frac{1}{g(\varphi)} d\varphi \right)
\]

Therefore, the winding function for winding \( x \) is:

\[
N_x(\varphi) = n_x(\varphi) - \frac{\int_0^{2\pi} \frac{n_x(\varphi)}{g(\varphi)} d\varphi}{\frac{1}{g(\varphi)}}
\]

Thus, the mutual inductance of winding \( x \) and \( y \) is:

\[
L_{yx} = \int_0^{2\pi} P(\varphi) N_x(\varphi) n_y(\varphi) d\varphi
\]

This modified winding function is valid for a non-linear airgap. In [8] and [9], different winding function has been used which is valid only for the uniform airgap. Thus, application of such winding function leads to unequal \( L_{xy} \) and \( L_{yx} \) (mutual inductance between stator and rotor). However, when the permeability over different media is independent of the position, such inequality is held [10]. The winding function introduced in this paper satisfies the equality \( L_{xy} = L_{yx} \), because substituting (28) in (29) leads to the following equation of the mutual inductance with the cumulative feature:

\[
L_{yx} = \int_0^{2\pi} P(\varphi) n_x(\varphi) n_y(\varphi) d\varphi
\]

5. Calculation of Inductances

Different inductances of the motor can be calculated based on (30) and determination of the winding turn functions. If the linear rise of mmf in the airgap above the stator and rotor slots is taken into account, the turn functions of the first stator phase and the first loop of the rotor are evaluated for the proposed motor as shown in Fig. 3. The turn functions of remaining phases and loops are obtained by \( 2\pi/3 \) displacement of these two functions. The inductances of the motor are then evaluated using the turn functions. Fig. 4 demonstrates different inductances of rotor and stator windings of a typical motor under the static, dynamic and mixed eccentricities conditions. It is clear that the stator inductances are independent of the rotor angular position under static eccentricity, but they dependent on the rotor angle.

**Fig. 3.** Turns function of: Left to right: first phase of stator, first loop of rotor for other two eccentricity modes. On contrary, the rotor inductances are independent of the rotor angular position in the dynamic eccentricity but the stator inductances depend on the angle. All rotor and stator inductances depend on the rotor angular position under the mixed eccentricity condition.
As shown in Fig. 4, in case of dynamic eccentricity condition the self-inductance of phase A is equal to the self-inductance of phase C, mutual inductance between phase A and B is equal to that of phase B and C. The reason for this equality is the stator phases winding. Thus, in general case, this may not be satisfied.

![Figure 4: Different inductances of motor under different eccentricities](image)

Fig. 5 shows the mutual inductance between the stator first phase and the rotor first loop under different eccentricity conditions. In order to confirm the inductance calculation using the...
modified winding function, finite element computations have been compared to those calculated analytically as shown in Fig.6.

![Fig. 5. Mutual inductances of stator first phase and first loop of rotor in different eccentricities](image)

![Fig. 6. Mutual inductances between stator phases (Left: phase A, Right: phase B) and rotor loop1, obtained using winding function theory and finite element method with 15% static and 5% dynamic eccentricity](image)

6. Simulation Results

Simulation of a three-phase squirrel-cage induction motor under different eccentricity, particularly the mixed case, is very complicated. At this end, matrices of the stator and rotor inductances as well as the mutual inductances and their derivatives must be calculated and substituted in the electromagnetic and electromechanical equations of the motor. These equations have a very large dimensions and their solution takes too long.

Performance of motor in time domain and under mixed eccentricity condition is compared with that of the healthy motor in Fig.7. However this comparison isn't useful for the faults diagnosis.

MCSA theory suggests the comparison of the frequency spectra of the lines current. Fig. 8 compares the line current frequency spectrum of the dynamic eccentricity with the healthy motor.

As Fig. 8 demonstrates, there are two sidebands around main frequency in case of mixed eccentricity condition. Also around each of Principal Slot Harmonics \( (f_{PSH}) \) two additional harmonics are amplified in mixed eccentric conditions. Frequencies of these harmonics are calculated with (31). Existence of these harmonic is a good benchmark for confirmation of our simulation while it is approved by experimental results.

\[
\begin{align*}
\delta_{ecc} &= 0.3 \\
\delta_s &= 0.3 \\
\delta_e &= 0.2, \delta_d = 0.1
\end{align*}
\]

\[
\begin{align*}
f_{ecc} &= \left\{ \begin{align*}
\left(1 \pm \frac{1-s}{p}\right) f_0 = f_0 \pm f_{m} \\
\left(1 \pm \frac{1-s}{p}\right) f_0 = f_{PSH} \pm f_{m}
\end{align*}\right. \\
f_{ecc-PISH} &= \left\{ \begin{align*}
\left(1 \pm \frac{1-s}{p}\right) f_0 = f_{PSH} \pm f_{m}
\end{align*}\right.
\]

The most usual procedure for the eccentricity diagnosis is detection of these harmonics.
7. Conclusions

The winding function in the eccentricity has been modified. This led to a more comprehensive winding function of different windings. Using this function satisfies the condition $L_{xy}=L_{yx}$, while some papers violate this condition and incorrect winding function has been employed for analysis of three-phase squirrel-cage induction motor under static and dynamic eccentricity. In addition, a more accurate approximation has been used for airgap permeance evaluation under eccentricity that leads to more accurate calculated inductances.

Variation of the calculated inductances under different eccentricity has been illustrated. Simulation has shown that the harmonics characteristic under eccentricity has amplified the line current frequency spectrum. Any healthy motor has slight eccentricity and thus the current frequency spectrum but it is amplified under significant eccentricity.

8. List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$[Vs]$</td>
<td>stator voltage vector</td>
</tr>
<tr>
<td>$[Is], [Ir]$</td>
<td>stator and rotor current vectors</td>
</tr>
<tr>
<td>$[\psi_s], [\psi_r]$</td>
<td>stator and rotor flux vectors</td>
</tr>
<tr>
<td>$[R_s], [R_r]$</td>
<td>stator and rotor resistance matrices</td>
</tr>
<tr>
<td>$r_0, g_0$</td>
<td>symmetrical mean radius and length of airgap</td>
</tr>
<tr>
<td>$g(\phi)$</td>
<td>airgap length distribution</td>
</tr>
<tr>
<td>$rav(\phi)$</td>
<td>airgap average radius distribution</td>
</tr>
<tr>
<td>$\delta$</td>
<td>mixed eccentricity degree</td>
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</table>
stator and rotor inductance matrices

\[ \begin{bmatrix} L_s & L_{sr} \\ L_{sr} & L_r \end{bmatrix} \]

\[ \alpha \] mixed eccentricity angle

\[ n \] number of rotor bars

\[ T_l, T_m \] load and motor torques

\[ \theta, \omega_r \] angular position and speed

\[ R_s, R_r \] inner radius of stator and rotor

\[ r_s \] stator phase resistance

\[ r_b, r_e \] rotor bar and ring resistances

\[ [V_s] \] stator voltage vector

\[ \phi \] arbitrary angle of stator reference frame

\[ P(\phi) \] magnetic permeance distribution of airgap

\[ H_s(\phi,r) \] magnetic field intensity of winding x

\[ J_s, i_s \] current density and current of winding x

\[ n_s(\phi) \] turns function of winding x

\[ N_s(\phi) \] winding function of winding x

\[ L_{xy} \] mutual inductance between winding x and y

\[ f_0 \] supply fundamental frequency

\[ f_{em} \] No. of rotor rotation in one second

\[ p \] Number of pole pairs

9. References


Appendix (simulated motor specifications)

Output power: 11 kW
L: 11 cm
R: 8.2 cm
Rotor: squirrel-cage
Number of group per phase: 2
Number of winding turns per phase group: 4
Number of turns per winding: 28
Number of rotor bars: 40
Number of stator slots: 48